Guess the answers.
Do these integrals converge or diverge?

Answers

	Converge	Div
① $\displaystyle\int_3^\infty \frac{x^5+e^x+2}{x^{10}-2x}\,dx$	7	2
② $\displaystyle\int_3^8 \frac{\sqrt{x^8}}{\sqrt{x^{11}}+2x+\sin(x)}\,dx$	15	0
③ $\displaystyle\int_1^4 \frac{1+\sqrt{x}}{(x-2)^2}\,dx$	9	2
④ $\displaystyle\int_{-1}^7 (x-2)^{-2/7}\,dx$	7	7
⑤ $\displaystyle\int_2^\infty \sin(x)+\frac{1}{x^8}\,dx$	3	12
⑥ $\displaystyle\int_0^{2\pi} \sin(x)+\frac{1}{x^8}\,dx$	6	12

① fcn $\to\infty$

② x^4 Not improper.

③ $= \infty$; near $x=2$ $\approx \frac{1+\sqrt{2}}{(x-2)^2}$; $\int_0^1 \frac{1}{x^2}\,dx = \infty$

④ near $x=2$ $\frac{1}{(x-2)^{2/7}}$ like $\int_0^1 \frac{1}{x^{2/7}}\,dx$

$= \int_{-1}^{2} + \int_2^7 \frac{1}{(x-2)^{2/7}}\,dx$

$u = x-2$, $du = dx$ $\to \int_0^5 \frac{1}{u^{2/7}}\,du$

⑤ $\to$ oscillates; $\int_2^\infty \frac{1}{x^8}\,dx$ converge $p=8>1$

⑥ oscillates. diverges $\to\infty$ near 0

(7) $$\int_0^2 \frac{\sqrt{x}}{\sin(x)}\,dx \quad \frac{c}{5} \quad \frac{d}{8}$$

$\approx \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$ near $x=0$.

Near $x=0$

$\frac{\sin(x)}{x} \approx 1$

$\sin(x) \approx x$

(8) $$\int_{-\infty}^{-1} \frac{e^x + 7}{|x|^{1.001}}\,dx \quad \frac{c}{2} \quad \frac{d}{11}$$

near $x \to -\infty$

$\approx \frac{7}{|x|^{1.001}}$ Converges

$p = 1.001 > 1$

(9) $$\int_{-\infty}^{\infty} \frac{1}{x^2+2}\,dx \quad \frac{c}{7} \quad \frac{d}{10}$$

problem at $x \to \infty$, $x \to -\infty$ $\approx \frac{1}{x^2}$ $p=2$ conv.

(10) $$\int_{-\infty}^{\infty} \frac{x^3 - 3x^2 + 2}{x^4 + 7x + 2}\,dx \quad \frac{c}{11} \quad \frac{d}{2}$$

No problems except

at $x \to \infty$, $x \to -\infty$ $\Big\}$ for $x \to \pm\infty$

$\approx \frac{1}{x}$ $p=1$ diverges.

(11) $$\int_{-\infty}^{\infty} \frac{x^3 - 3x^2 + 2}{x^6 + 7x + 2}\,dx$$

$\approx \frac{1}{x^3}$ near $x = \pm\infty$

converge.

$$\int_0^{\infty} \frac{x^3 - 3x^2 + 2}{x^5}\,dx$$

$\approx \frac{2}{x^5}$ near $x = 0$

diverges $\rightarrow \infty$ area

$p = 5 > 1$

near $x \to \infty$

$\approx \frac{1}{x^2}$ converge.

$p = 2$

∞ area.

Examples: Converge or diverge?

① $\displaystyle\int_2^{\infty} \frac{\sqrt{x} + 2^{-x} - \cos(x)}{x^2}\,dx$ $\quad \frac{c}{6} \quad \frac{d}{8}$

$x \to \infty \approx \frac{x^{1/2}}{x^2} = \frac{1}{x^{3/2}}$ $p = \frac{3}{2}$

② $\displaystyle\int_2^{\infty} \frac{\sqrt{x} + 2^{-x} - \cos(x)}{x}\,dx$ $\quad \frac{c}{1} \quad \frac{d}{14}$

$x \to \infty \approx \frac{1}{x^{1/2}}$ $p = \frac{1}{2}$

③ $\displaystyle\int_0^{2} \frac{\sqrt{x} + 2^{-x}}{x}\,dx$ $\quad \frac{c}{7} \quad \frac{d}{5}$

near $x = 0 \approx \frac{1}{x}$ $p = 1$ diverge more formal solution: for $0 \le x \le 2$ $\quad \frac{1}{4} = 2^{-2} \le 2^{-x} \le 2^{0} = 1$

$$\frac{1/4}{x} \le \frac{\sqrt{x} + 2^{-x}}{x} \le \frac{\sqrt{x} + 1}{x} \le \frac{\sqrt{2}+1}{x}$$

and $\frac{1}{4}\int_0^2 \frac{1}{x}\,dx = \infty$

$$\therefore \int_0^2 \frac{\sqrt{x} + 2^{-x}}{x}\,dx = \infty$$

Geometric Applications of the integral!

$\boxed{\text{Ex}}$ Find the area between the curves $y = x^2 - 3$ and $y = 2x$.

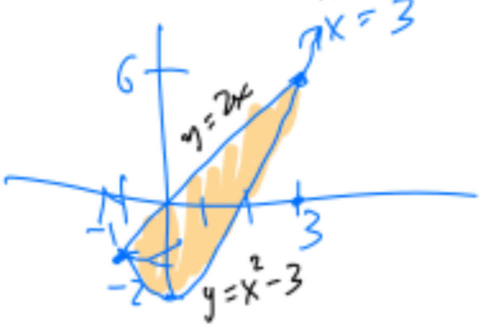
① Intersection pts $\quad x^2 - 3 = 2x$

$$x^2 - 2x - 3 = 0$$

$$(x+1)(x-3) = 0$$

$x = -1$

$x = 3$



② $\text{Area} = \int_{-1}^{3} (\overset{\text{top}}{f(x)} - \overset{\text{bottom}}{g(x)})\,dx$

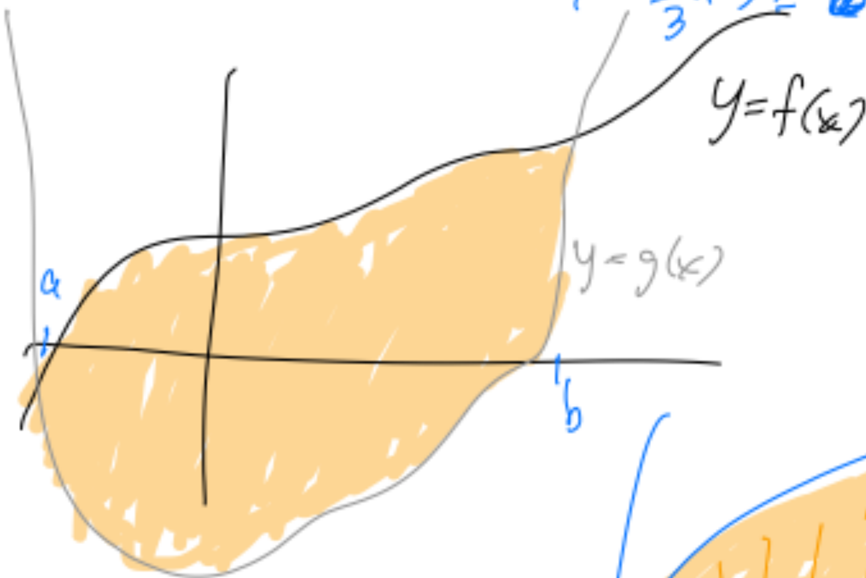
$$= \int_{-1}^{3} (2x - (x^2 - 3))\,dx$$

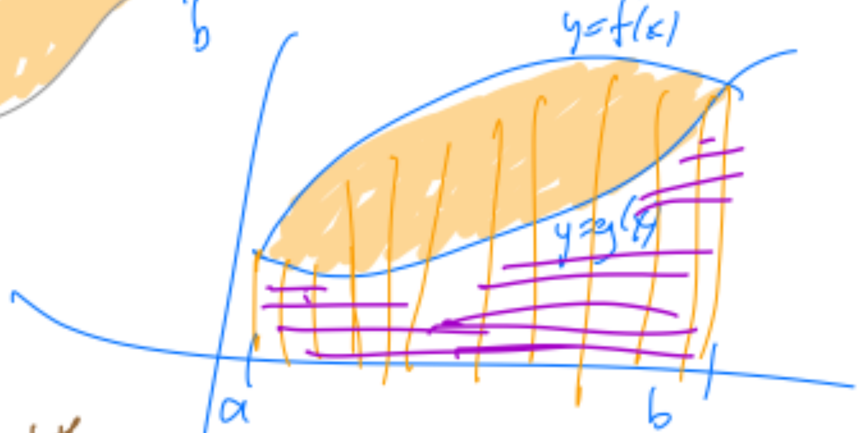
$$= x^2 - \frac{x^3}{3} + 3x \Big|_{-1}^{3}$$

$$= (9 - 9 + 9) - \left(+1 - \frac{-1}{3} - 3\right)$$

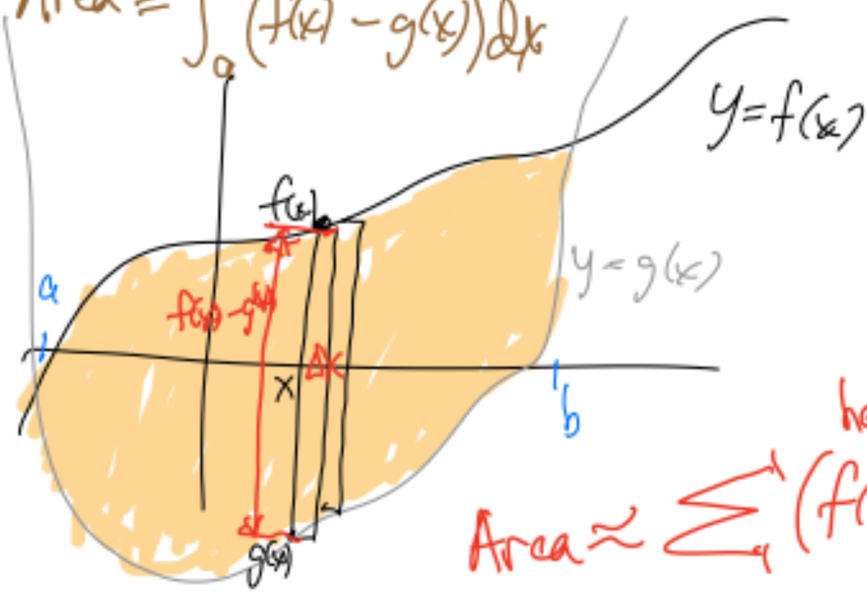
$$\frac{4}{3} - 3$$

$$= 9 - \frac{4}{3} + 3 = \frac{36 - 4}{3} = \boxed{\frac{32}{3}}$$





$$\text{Area} = \int_a^b (\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}})\,dx$$



$$\text{Area} \approx \sum \underbrace{(f(x) - g(x))}_{\text{height of a rectangle}} \underbrace{\Delta x}_{\text{width}}$$

$$\begin{matrix}\text{exact} \\ \text{area}\end{matrix} = \lim_{\Delta x \to 0} \left(\quad " \quad \right)$$

$$= \int_a^b \left(f(x) - g(x)\right) dx.$$